New Method for the Analysis of Dispersion Characteristics of Various Planar Transmission Lines with Finite Metallization Thickness

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Abstract—**It is shown that the eigen-function weighted boundary integral equation method can be extended to analyze the dispersion characteristics of various planar transmission lines with finite metallization thickness, such as microstrip lines, conductor-backed coplanar waveguides, and micro-coplanar strip lines. The computational results clearly demonstrate the effect of finite strip thickness on the propagation properties of these transmission lines for MMIC's.**

I. Introduction

UNTIL NOW, a great deal of theoretical methods have been presented for analyzing the propagation properties of various planar transmission lines with negligible finite metallization thickness [1]. With the consideration of practical application for MMIC's, some numerical analysis methods have been proposed recently to study the effect of the strip thickness [2]–[6], and provided the calculated results at low frequencies. However, these papers have not revealed the metallization effect at high frequencies, which is a real problem to be necessarily solved theoretically.

In the early 1980's, the boundary element method (BEM) began to be applied for the analysis of electromagnetic boundary-value problems [7]. In the BEM, the boundary integral equations are set up from the Green's identity by using the free-space Green's function as the weighted function. Unfortunately, the BEM may lead to singularity problems and computational time problems for regular boundary conditions in the electromagnetic field. In the analysis of the uniform-core optical fibers, Kishi *et al.* [8] have presented the boundary integral method without using Green's function, in which the eigen-function was firstly used instead of Green's function in order to overcome the singularity. At the almost same time, Zhu *et al.* [9], [10] also proposed the eigen-function weighted boundary integral equation method (EW-BIEM) to analyze the propagation properties of arbitrary cross-sectional metal waveguides and grooved nonradiative dielectric waveguides, in which the eigen-function satisfying the regular boundary conditions was employed instead of Green's function, so that the computational time was decreased to a great extent and the singularity problems from the Green's function was also avoided.

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II. Analysis Method

In this letter, the EW-BIEM method is applied effectively to accurately analyze the dispersion characteristics of various planar transmission lines with finite metallization thickness, such as microstrip (MS) lines, conductor-backed coplanar waveguides (CPW), and micro-coplanar strip (MCS) lines as shown in Fig. 1. Compared with the BEM, the prominent advantages of this method are that all of the boundary integral equations can be set up only on the surfaces of the finite strips and the interfaces between air and dielectric subregions but not on all closed surfaces. This is because both the field functions and eigenfunctions satisfy the regular homogeneous boundary condition, and the effect of the edge on the strip can easily be handled theoretically.

For the convenience of analysis, a sufficiently large metal box is firstly added to the outer section of transmission lines at a far place from the central strip. Then the two-dimensional Green's identity of second kind is utilized to set up the boundary integral equations in air and dielectric subregions. Since both of field quantities and eigenfunctions satisfy the Hemholtz's equation, the area integral term disappears naturally and only the boundary integral term in the following form remains

$$\oint_{\Gamma}\left[W_{(r)}\frac{\partial \Phi_{(r)}}{\partial n}-\Phi_{(r)}\frac{\partial W_{(r)}}{\partial n}\right]d\Gamma=0, \tag{1}$$

where $\Phi_{(r)}$ is the field function to be solved, $W_{(r)}$ is the eigen-function, and Γ denotes the closed surface of the considered region.

For MS lines and CPW, only a half of cross-section should be analyzed considering the structural symmetry. According to the rectangular homogeneous metal boundary conditions, four analytical eigen-functions corresponding to TM-mode and TE-mode in the two subregions can be expressed directly in the simple multiplied form of sine and cosine functions. By substituting these eigen-functions subsequently into (1) and utilizing the continuity conditions of tangential field components in the interface between the two subregions, a set of

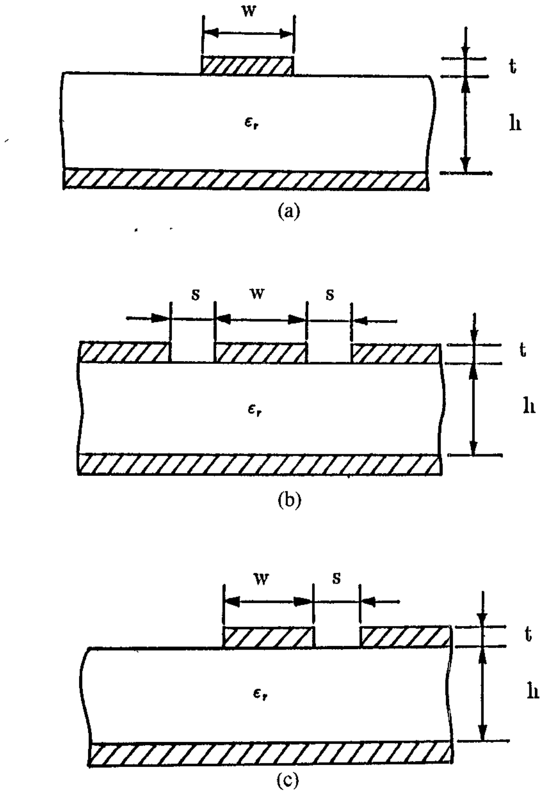

Fig. 1. Planar transmission lines with finite metallization thickness t. (a) MS lines. (b) Conductor-backed CPW. (c) MCS lines.

four coupled boundary integral equations can be easily constructed.

$$\begin{cases} \int_{\Gamma_i}\left[E_z\frac{\partial W_I^e}{\partial n} - W_I^e\frac{\partial E_z}{\partial n}\right]d\Gamma - \int_{\Gamma_{S_1}}\left[W_I^e\frac{\partial E_z}{\partial n}\right]d\Gamma = 0, \\ \int_{\Gamma_i}\left[H_z\frac{\partial W_I^h}{\partial n} - W_I^h\frac{\partial H_z}{\partial n}\right]d\Gamma + \int_{\Gamma_{S_1}}\left[H_z\frac{\partial W_I^h}{\partial n}\right]d\Gamma = 0, \\ \int_{\Gamma_i}\left[E_z\frac{\partial W_{II}^e}{\partial n} - W_{II}^e\frac{\partial E_z}{\partial n}\right]d\Gamma - \int_{\Gamma_{S_2}}\left[W_{II}^e\frac{\partial E_z}{\partial n}\right]d\Gamma = 0, \\ \int_{\Gamma_i}\left[H_z\frac{\partial W_{II}^h}{\partial n} - W_{II}^h\frac{\partial H_z}{\partial n}\right]d\Gamma + \int_{\Gamma_{S_2}}\left[H_z\frac{\partial W_{II}^h}{\partial n}\right]d\Gamma = 0, \end{cases} \tag{2}$$

where, Γ_i denotes the air-dielectric interface; Γ_{S_1} and Γ_{S_2} the above and bottom surfaces of the metal strip, respectively; I and II denote the air and dielectric subregions, respectively; e and h mean TM- and TE-modes, respectively.

However, it is difficult to treat the effect of a sharp strip conductor edge because the field distributions on the 90° conductor edge is seriously changeable as a result of its strong sensitivity to the computational process and the difficulty in the convergence of calculated propagation constant [11]. So, we treat the 90° edge as the 90° circular arc of infinitesimal radius r, as shown in Fig. 2, to avoid the singularity of field derivatives. In practice, it is the integral value of the field in the 90° arc that takes a real role in the boundary integral equations. Therefore, if the multiplied form of the field quantity and radius would be chosen as the unknown function in the 90° arc, the effect of such 90° edge can be considered in each integral equation easily.

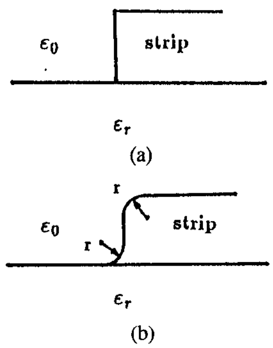

Fig. 2. Consideration of 90° edge in the strip. (a) 90° edge. (b) 90° circular arc of infinitesimal radius r.

With the discretization of the boundary integral path in each boundary integral into finite segments, a set of homogeneous linear equations can be obtained from (2) as

$$[H(\beta)][X] = [0]. \tag{3}$$

Based on the condition for nontrivial solutions, the propagation constant β or effective dielectric constant ϵ_{eff} can be accurately obtained as the numerical solution of the determinantal equation,

$$\det[H(\beta)] = 0. \tag{4}$$

III. Numerical Results

Fig. 3(a) shows the comparison of our results with Shih's [10] based on the variational conformal mapping technique for MS lines with finite metallization thickness $t = 0.0001$, 0.2, and 0.5 mm. Both of the two results have clearly demonstrated the influence of the metallization thickness on the values of effective dielectric constants, especially at low frequencies. However, it is also found that the two results still indicate the discrepancy of less than 4%, which may be due to the approximation by Shih using a conformal mapping technique. In the numerical calculations for the MS lines, it is found that taking 24 discretization points in total for dielectric interfaces and strip surfaces ensures the accuracy of less than 0.5% and typical computation time for one frequency point is about 2 minutes on a Sun 4 workstation.

Fig. 3(b) and (c) show the calculated effective dielectric constants of conductor-backed CPW and MCS lines, with the different strip thickness $t = 0$, 2 and 5 μm, by using this method. As for the planar transmission lines of MMIC's with the small structural dimensions, it is found from the calculated results that the thickness of conductor strip would affect the propagation properties seriously. With the increase of the strip thickness, the effective dielectric constant will be decreased principally. Therefore, it is practically necessary to consider the effect of strip thickness in the analysis of the transmission lines and components of MMIC's.

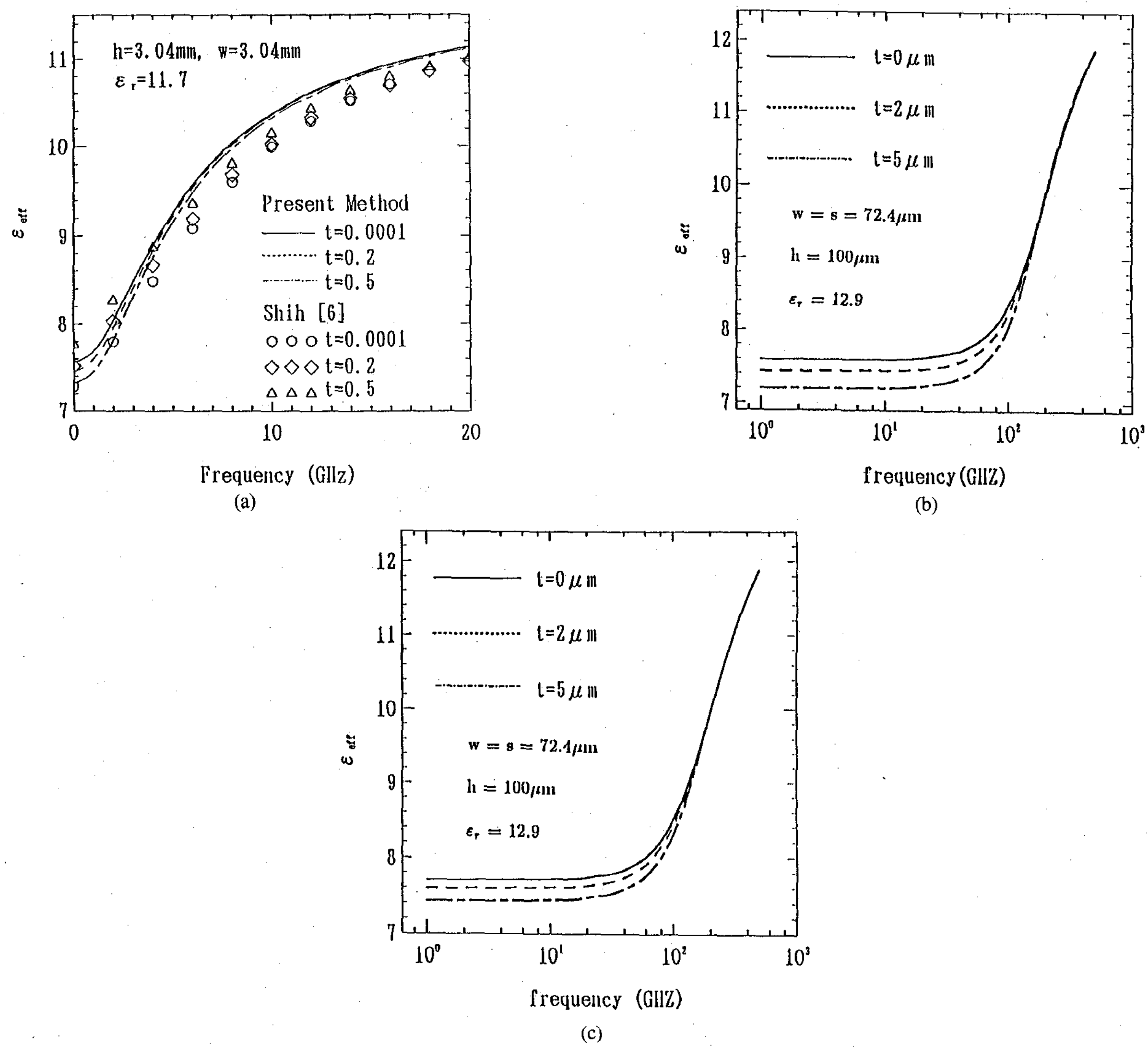

Fig. 3. Dispersion characteristics of planar transmission lines with finite metallization thickness t. (a) MS lines. (b) Conductor-backed CPW. (c) MCS lines.

IV. Conclusion

In this letter, we present a new boundary integral equation method with the weighted function of the simple eigenfunction instead of the complicated Green's function, to be applied to the analysis of the propagation properties of general planar transmission lines with finite metallization thickness, such as, microstrip lines, conductor-backed coplanar waveguides, and micro-coplanar striplines. A series of dispersive curves over a wide frequency range have been provided to show the effect of the strip thickness on the propagation properties. This method could be applied to the analysis of more complicated transmission lines and discontinuity.

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